Math 304 (Spring 2015) - Homework 8

Problem 1.

Determine whether the following sets of vectors form an orthonormal basis of $\mathbb{R}^2$.

(a) $\{(1,0)^T, (0,1)^T\}$

(b) $\left\{ \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right)^T, \left(\frac{-1}{2}, \frac{\sqrt{3}}{2} \right)^T \right\}$

(c) $\left\{ \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \right\}$

Problem 2.

Let $\{u_1, u_2, u_3\}$ be an orthonormal basis for an inner product space V and let

$$w = u_1 + 2u_2 + 2u_3 \quad \text{and} \quad v = u_1 + 7u_3$$

Determine the value of each of the following:

(a) $\langle w, v \rangle$

(b) $\|w\|$ and $\|v\|$

(c) the angel between w and v.

Problem 3.

Given the basis $\{(1,2,-2)^T, (4,3,2)^T, (1,2,1)^T\}$ for $\mathbb{R}^3$, use the Gram-Schmidt process to obtain an orthonormal basis.

Problem 4.

Let

$$A = \begin{pmatrix} 3 & -1 \\ 4 & 2 \\ 0 & 2 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 0 \\ 20 \\ 10 \end{pmatrix}.$$

(a) Find an orthonormal basis of the column space of A.

(b) Find the projection of v onto the column space of A.

Problem 5.

(**Legendre Polynomials**) Let $\mathbb{P}_2 = \{$all polynomials of degree $\leq 2\}$. We define the following inner product on $\mathbb{P}_2$:

$$\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx.$$

Start with a basis $\{1, x, x^2\}$ of $\mathbb{P}_2$, use the Gram-Schmidt process to obtain an orthonormal basis.